THE FIELD IONIZATION CHARACTERISTICS
OF INDIVIDUAL ATOMIC PLANES

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A detailed study of the field ionization characteristics of nine different planes of tungsten in the (001)-(011) standard triangle has been performed as a function of tip temperature (Tt) between 11 K and 86 K, and local radius of curvature (r). The measured helium ion current–voltage characteristic curve for each plane was found to consist of two distinct regimes. The first regime was linear on a log–log plot, and the slope varied from 27 to 41. A modified version of Gomer's model for the very low field ion current fitted the regime I data reasonably well. The second regime of each characteristic curve was quite complicated and exhibited several maxima and minima whose positions were functions of both Tt and crystallographic plane. A qualitative explanation for the behavior of the ion current in regime II was given in terms of a patch field model consisting of three dominant spatial regions on the surface of the field ion microscope specimen. In addition, the explanation also considered the role played by a lateral supply of gas atoms on the specimen's surface, and a slowly increasing field dependent radial supply function of gas atoms. It was also found that the probability of ionization was a strong function of both Tt and crystallographic plane. An expression was derived for the temperature dependence of this effect which fitted the data for atomically smooth planes [e.g., the (011) plane]. Finally, the ion current from individual planes was proportional to a power of r (at constant electric field) which varied between 2.3 and 2.9. This result was at variance with the existing theories of the supply function, and indicated that the shank of the specimen was a significant source of imaging gas atoms.

1. Introduction

The measurement of ion current as a function of electric field (Et) offers one of the few available tests of theories pertaining to the field ionization processes occurring near a field ion microscope (FIM) tip [a recent review of this subject may be found in chapter II of Müller and Tsong's book1]. To date, a number of studies3–8 has been performed in which the total ion current from a FIM tip was measured as a function of Et in various imaging gases over a range of tip temperature (Tt) between 15 K and room temperature.

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The effect of increasing tip voltage \( (V_T) \) at constant \( T_T \) on the field ionization characteristics of individual crystal planes can be seen from an examination of the six micrographs shown in fig. 1. These micrographs are of a tungsten specimen which was field evaporated to an end form at 36°C, and then imaged at 11°C. It is clear from these micrographs that the ion current as a function of \( V_T \) differs markedly from one crystallographic plane \([hkl]\) to another. Hence, the total ion current measurements sum the contributions from the different \([hkl]\) planes, but do not reveal the very strong dependence of the ion current on a given \([hkl]\) plane. The experimental studies of the field ionization characteristics of individual planes have been limited. Müller et al.\(^9\) measured the brightness of the (111) region of tungsten photometrically as a function of \( V_T \) at 4.2°C, 21°C and 78°C. The only study of the \([hkl]\) dependence of ion current \( i([hkl]) \) is the one by Plummer and Rhodin\(^10\) who measured the ion current as a function of \( E_T \) for the (111), (100), (112), and (110) planes of tungsten at 78°C. In addition, Tamaki and Sugata\(^11\) photometrically measured the image intensity of the (001), (011) and (111) planes of W and Mo specimens as a function of \( E_T \) at 78°C. The above three papers constitute the total literature on the subject of characteristic curves as a function of \([hkl]\) plane and \( T_T \).

In the present paper we present a detailed study of the \( i([hkl]) \) versus \( V_T \) characteristic curve of a number of individual atomic planes of tungsten in the \( \langle 001 \rangle - \langle 011 \rangle - \langle 111 \rangle \) standard triangle as a function of \( T_T \) (11°C to 86°C), and local radius of curvature \( (r_T) \). These curves were found to have a strong dependence on both \( T_T \) and \([hkl]\) plane. The experimental results indicate that the existing theories of field ionization, and the supply function are not adequate for explaining the observed phenomena quantitatively\(^12\).

2. Experimental techniques

2.1. General techniques

The FIM employed was an all stainless steel microscope which was operated under static vacuum conditions with a background pressure in the range \( (1 \text{ to } 10) \times 10^{-10} \text{ Torr} \). The quantity \( T_T \) was varied via a modified version of a continuous transfer liquid helium cryostat [Seidman et al.\(^13\)] which allowed \( T_T \) to be controlled to a stability of \( \pm 0.01^\circ \text{K} \). Seidman and Scanlan\(^14\) have shown that heating effects in the tip were negligible, hence the measured temperatures should correspond to \( T_T \) very well. At \( T_T \) equal to 12°C the

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Fig. 1. A series of FIM patterns of tungsten taken at 11°C showing the effect of increasing \( V_T \) on the image contrast. The rectangular black region at \( \approx 10 \) o'clock is the shadow of the Faraday cup. The (011) plane is at the center of the micrographs.
temperature of the radiation shield surrounding the specimen was at 34 °K, therefore, the temperature of the imaging gas in the region around the specimen was somewhere between these two temperatures. As $T_T$ was increased the temperature of this region increased, so that the thermal energy of the gas in this region was a function of $T_T$. The temperature of the gas at appreciable distances from the tip was room temperature. Further details regarding the FIM, temperature measurement, and specimen preparation have been reported by the authors\textsuperscript{15} previously.

The quantity $V_T$ was controlled with a zero to 30 kV Spellman power supply (Model RHR30PN). The measured combined regulation and ripple of this unit was 0.003%, the short term stability was better than 0.01% over a 15 min period, and the long term stability was better than 0.05% over an 8 h period. The voltage was measured with a Hewlett-Packard 4-place digital voltmeter used in conjunction with a 100:1 voltage dividing network. The parameter $i[(hk)]$ was measured with a Keithley 602 electrometer in combination with a specially constructed Faraday cup.

2.2. THE FARADAY CUP

A schematic diagram of the Faraday cup is shown in fig. 2. The helium ion current was collected on a stainless steel flag which was spot welded to a tungsten rod that was part of a glass-to-metal seal. The shield of the Faraday cup was stainless steel, and was maintained at earth potential. This arrangement allowed reproducible measurements of $i[(hk)]$ in the $10^{-15}$ A range.

The size of the aperture in the Faraday cup shield, and the distance from the tip to the collection flag were such that an area of $\approx 1.8 \times 10^{-14}$ cm$^2$ was examined on the surface of the specimen. The $(hk)$ plane studied was determined by using the positioning hair (see fig. 2) on the Faraday cup shield, and a comparison of FIM micrographs with and without the cup in position. The vertical and horizontal adjusting screws were designed so that the Faraday cup could be moved in and out of a given position without any significant mechanical hysteresis.

The characteristic curves were measured with ascending and descending $V_T$ to check for hysteretic effects, and curves which exhibited hysteresis were rejected. Finally, we note that no special precautions were taken to eliminate secondary electrons, hence the values of $i[(hk)]$ reported are not absolute currents. Many of the maxima and minima recorded on the characteristic curves were also visually observed, hence, it is felt that the secondary electrons did not have a strong effect on the shape of these curves even though they affect the absolute values of $i[(hk)]$.

3. Experimental results

The results in this section are presented in 3 sub-sections. The first sub-section (3.1) gives the effect of $(hk)$ plane on the characteristic curve at 3 different values of $T_T$ (11 °K, 36 °K, and 78 °K). The second sub-section (3.2) presents the effect of $T_T$ on the characteristic curve for 3 different $(hk)$ planes $(111), (233)$ and $(011)$. The third sub-section (3.3) gives the dependence of $i[(hk)]$ on $r_T$.

3.1. THE EFFECT OF $(hk)$ PLANE ON THE $i[(hk)]$ VERSUS $V_T$

The characteristic curve was measured at 11 °K, 36 °K, and 78 °K (see figs. 3 to 5) for the $(233), (111), (013), (123), (334), (112), (114), (001)$, and $(011)$ planes. The experiments were performed by first field-evaporating a specimen to an end form at $36^\circ$K, and then measuring all of the characteristic curves at each value of $T_T$ for all the planes. At each $T_T$ the evaporation field was not exceeded, hence for the curves at a given $T_T$ the values of $V_T$ correspond to values of $E_T$. The characteristic curves in figs. 3 to 5 exhibited the following general features:

(1) Each curve consisted of 2 distinct regimes. The initial regime (number I) was one of constant slope where $i[(hk)]$ showed a strong dependence on $V_T$. The values of $i[(hk)]$ were summed to form a total ion current versus $V_T$ curve, and the slopes of these curves in regime I were then calculated. These slopes were found to be 38, 35, and 32 at $T_T$ equal to 11 °K, 36 °K, and 78 °K respectively; i.e., the slope increased with decreasing $T_T$. The second regime (number II) corresponded to the range of $V_T$ in which individual atoms were
resolved, and showed a much weaker dependence on $V_T$. Regime II is the portion of the total ion current characteristic curve which Southon and Brandon\textsuperscript{5} have denoted the "working range" of a FIM.

(2) The curves exhibited maxima and minima in regime II, which occurred at different values of $V_T$ for different $(hkl)$ planes.

(3) The shape of the curves showed a marked dependence on $T_T$. In particular, the effect of increasing $T_T$ was to smooth out the shape in regime II, and to increase the value of $V_T$ at which field ionization first occurred. (The limited data of Müller et al.\textsuperscript{9} for the (111) plane of tungsten exhibited a similar effect.)

(4) The emitting surface was made up of approximately three different
3.2. The Effect of $T_T$ on the $i([hkl])$ versus $V_T$ curves for the (111), (233) and (011) Planes

Once again the specimen employed was first field evaporated to an end form at 36°K, and then the characteristic curve was measured at 11°K, 23°K, 36°K, 49°K, 62°K, 74°K, and 86°K for a given plane without changing the position of the Faraday cup. Since $V_T$ was never raised to the point where field evaporation occurred, the quantity $E_T$ was the same for each curve at the various $T_T$. The curves shown in figs. 6 to 8 had the following characteristics:

1. The value of $V_T$ at which a measurable value of $i([hkl])$ was first obtained decreased as $T_T$ was decreased.
2. The cut-off voltage ($V_c$) decreased as $T_T$ was decreased (see sub-section 4.1 for a definition of $V_c$). For the (111) plane this decrease was 30.5% for a $\Delta T_T$ of 75°K, for the (233) plane the decrease was 25.4% for a $\Delta T_T$ of 63°K, and for the (011) plane the decrease was 6.4% for a $\Delta T_T$ of 51°K.

<table>
<thead>
<tr>
<th>Plane (hkl)</th>
<th>Temperature (°K)</th>
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<tbody>
<tr>
<td>(111)</td>
<td>11</td>
</tr>
<tr>
<td>(233)</td>
<td>27</td>
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(3) The slopes of the curves in regime I were a function of $T_T$, but no systematic temperature dependence was found. The slopes for the (111) and (233) planes as a function of $T_T$ are given in table 1 which shows that this quantity had a range of values from 27 to 41.

(4) The (111) and (233) planes exhibited distinct maxima and minima in regime II. The height of the first maximum decreased as $T_T$ increased, and

* In this investigation no measurements were made of the ion current from zone lines, although it is clear that they contribute to the total ion current significantly at low fields. For example, in fig. 1 at 7.9 kV and 8.2 kV field ionization is occurring strongly above the zone lines as well as above the region A planes.
Fig. 6. The quantity $i(\{f\})$ as a function of voltage for the (111) plane at 7 different temperatures ranging from 11°K to 86°K. No field evaporation was performed in between the temperature changes, hence the values of $V_T$ correspond to the same electric field at each temperature. (This same comment also applies to figs. 7 and 8.)

by 62°K the first maximum disappeared completely leaving only a single maximum.

(5) The absolute magnitude of $i(\{f\})$ decreased as $T_T$ increased. The only exception to this statement occurred in the case of the (011) plane which showed a higher current at 23°K than at 11°K.

Fig. 7. The quantity $i(\{f\})$ as a function of voltage for the (233) plane at 6 different temperatures between 11°K and 74°K.

3.3. The Effect of $r_T$ on the $i(\{f\})$ Versus $V_T$ Characteristic Curves

The curves were determined for 2 specimens at 5 different values of $r_T$ between 200 Å and 450 Å. The general shape of the curves was independent of $r_T$ at a given $T_T$. The value of $i(\{f\})$ at constant $E_T$ was proportional to a power of $r_T$ between 2.3 and 2.9 for the (111) plane between 300 Å and
4. Discussion and interpretation of experimental results

4.1. A model for the temperature dependence of the cut-off voltage ($V_c$)

A striking feature of the experimental results presented in sub-sections 3.1 and 3.2 is the strong temperature dependence of $i((hkl))$. As one measure of this temperature dependence we defined the cut-off voltage ($V_c$) [Southon and Brandon].

![Graph showing $i((hkl))$ vs specimen voltage for different temperatures.](image)

**Fig. 8.** The quantity $i((hkl))$ as a function of voltage for the (011) plane at 5 different temperatures between 11°K and 62°K.

400 Å, and a power of 2.5 to 2.8 for the (011) plane in the range of $r_T$ between 230 Å and 400 Å. These measurements were made at various values of $E_T$ in regime II of the characteristic curves.

* The conversion from $V_T$ to $E_T$ was made using the expression $E_T = V_T/r_T$ where $\zeta$ is a constant which was taken to be 5. It was also assumed that $\zeta$ was independent of $r_T$.

![Schematic diagram of cut-off voltage](image)

**Fig. 9.** A schematic $i((hkl))$ versus specimen voltage characteristic curve which illustrates the method used to calculate the cut-off voltage ($V_c$).

Brandon] as the value of $V_T$ at which the curve in regime I deviated from a tangent line. The method for determining $V_c$ is schematically illustrated in fig. 9. As an example of this effect, a plot of $V_c$ as a function $T_T$ is shown in fig. 10 for the (111) plane at 5 different values of the average local radius of curvature. The average slope between 11°K and 86°K for the (111) plane was 26.7 V°K$^{-1}$, and the decrease in $V_c$ was a 30.5% effect for this $\Delta T_T$ of 75°K. This result indicates that $V_c$ is controlled by $T_T$. In a study of the characteristic curves of tungsten in the temperature range 63°K to 273°K (the total
ion current was measured) Southon and Brandon$^4$ also found $V_e$ to be a function of $T_T$. In addition, Brandon$^{17}$ presented a model to explain the temperature dependence of $V_e$ based on the principle that $T_T$ controls the velocity of the imaging gas atom when it is ionized*. In the following we reconsider and modify Brandon's derivation, and then apply it to our experimental data.

* Müller and Bahadur$^9$ originally calculated ionization probabilities, and assumed that the velocity of the gas atom when it was ionized was determined by the kinetic energy resulting from its polarizability. It is clear that if this were the case that $E_0$ would be independent of $T_T$. Hence the probabilities which they calculated do not correspond to the physical situation which is relevant to the field ionization process in the FIM.

Fig. 10. The cut-off voltage ($V_e$) as a function of $T_T$ for the (111) plane at 5 different values of the local radius of curvature ($r_2$) between 300 Å and 430 Å.

The model begins with a calculation of the characteristic lifetime ($\tau$) of a helium atom which is at a distance $x_e$ from the surface of a FIM tip. The quantity $\tau$ is given by the expression$^{1,18}$

$$\tau(x_e) = \left(\frac{\nu}{D}\right)^{-1}. \tag{1}$$

where $\nu$ is the frequency with which an electron attacks the deformed potential barrier of the electron, and $D$ is the transmission coefficient for an electron tunneling through the same barrier. The differential probability ($dP$) of an atom being ionized in an element of distance $dx$ which lies along a line that is orthogonal to the surface of the tip is given by

$$dP = \tau^{-1}(x_e) \, dx, \tag{2}$$

where $\nu$, is the velocity of a gas atom along this same line. The quantity $\nu$, can be obtained from the expression

$$\frac{1}{2}mv^2_e = \frac{1}{2}kT_T, \tag{3}$$

where $\gamma$ is a constant which depends on the degree of thermal accommodation of the imaging gas atom to $T_T$ when it is ionized (Chen and Seidman$^{19}$). Brandon assumed that $\gamma$ was equal to 1, and Van Eekelen$^{18}$ has calculated that the accommodation is almost complete at a specially defined $V_e^*$ (i.e., $\gamma \approx 1$). At the BIF characteristic of 115 K Chen and Seidman$^{18}$ measured values of $\gamma$ between 6.5 and 7.3 for pure helium as an imaging gas. Unfortunately, there are no experimental values of $\gamma$ at the cut-off field.

The quantity $dP$ increases as $T_T$ is decreased as a result of a reduced value of $\nu$. The quantity $D$ for a one-dimensional potential barrier is obtained from the WKB treatment (e.g., see Bohm$^{20}$) and is given by

$$\left\{ -2 \left(\frac{2m_e}{\hbar^2}\right)^{\frac{1}{2}} x_1 \right\} \int_{x_1}^{x_2} [V(x) - \varepsilon(x_e)]^{\frac{1}{2}} \, dx, \tag{4}$$

where $V(x)$ is the potential energy of an electron, $\varepsilon(x_e)$ is its total energy at $x_e$, $m_e$ the mass of an electron, and $\hbar$ ($\hbar = \hbar/2\pi$) is Planck's constant. The limits on the integral in eq. (4) are the classical turning points where $V(x) = \varepsilon(x_e)$.

All the ionization occurs in a narrow zone of width $\Delta x$ (Tsong and Müller$^{21}$), and the time ($\Delta t$) spent by an atom in traversing this zone is

* Van Eekelen$^{18}$ defines the cut-off field at the value of the electric field at which the gas concentration reaches a maximum. This definition is different from the one employed here, and is an inconvenient one from an experimental point of view.
given by
\[
\Delta t = \frac{\Delta x}{v_e} = \left( \frac{m}{\gamma k T} \right)^{\frac{1}{2}} \Delta x
\]  
(5)

The integrated probability [\(P(\Delta t)\)] of an atom being ionized during the
period \(\Delta t\) is given by the approximate expression\(^{18}\)
\[
P(\Delta t) \approx 1 - \exp \left( - \frac{\Delta t}{\tau} \right).
\]  
(6)

The value of \(P(\Delta t)\) at \(V_e\) depends on the relationship between \(\tau\) and \(\Delta t\),
hence we set \(\tau = \eta \Delta t\) where \(\eta\) is a number which is determined by the actual
value of \(P(\Delta t)\). Thus, one obtains the following relationship between \(D\) and \(T_T\)
\[
D \approx \frac{1}{\eta \Delta x} \left( \frac{\gamma k T_T}{m} \right)^{\frac{1}{4}}
\]  
(7)

Eq. (7) is a general expression which can be used to explain quantitatively
the dependence of \(E_e\) on \(T_T\) for any plane provided that \(V(x), \gamma, \) and \(\eta\) can
be specified.

We now proceed to interpret the observed variation of \(V_e\) (or \(E_e\)) with \(T_T\)
in terms of eq. (7).

4.1.1. The temperature dependence of the cut-off voltage of the (011) plane

To explain the dependence of \(E_e(V_e)\) on \(T_T\) we used two different forms for
\(V(x)\). The first form was given by the expression [Gomer\(^{18}\)]
\[
V_{(011)}(x) = -\frac{Ze^2}{x} + Ex
\]  
(8)

The first term in eq. (8) is the Coulomb attraction between the nucleus and
the electron, the second term is the work done on the electron by \(E\) (which is
assumed to be a constant independent of \(x\)), and the quantity \(Ze\) is the effective
charge on the nucleus. Following Gomer\(^{18}\) we replaced the potential
energy barrier described by eq. (8) by a triangle of height \((I_1 - 2(Ze^2E)^{\frac{1}{2}})\)
and base \((I_1 - \phi)/eE\), to obtain the following approximate expression* for
\(D_{(011)}\)
\[
D_{(011)} \approx \exp \left[-\left(\frac{2m_e}{\hbar^2}\right)^{\frac{1}{2}} \left(\frac{I_1 - \phi}{eE}\right) \left(I_1 - 2Ze^2E^{\frac{1}{2}}\right)^{\frac{1}{2}} \right],
\]  
(9)

where \(I_1\) is the first ionization potential of helium, and \(\phi\) the work function

It was assumed that the ionization process resulted in a singly charged ion.
4.1.2. The $E_s$ versus $T_T$ data for the (111) and (233) planes

The (111) and (233) planes of a bcc lattice are a good deal rougher on an atomic scale[29] than is the (011) plane. Therefore, one would not expect the potential $V_{011}$ or the image potential to give an adequate description of the electron potential energy in the ionization zone above the (111) and (233) planes. In an attempt to explain the experimental $E_s$ versus $T_T$ curves for these rougher planes we employed a potential described by Plummer[22]. His potential tried to take into account the variation in $E$ above a line of point charges. Unfortunately, it did not yield an $E_s$ versus $T_T$ curve with a large enough slope to explain the experimental results for the (111) and (233) planes.

4.2. Field ion current - Regime I

Regime I is the region of the characteristic curve which Gomer[15] has entitled the “Very Low Field” case. In this regime Gomer assumed that the probability of ionization was small enough, so that the concentration of gas atoms near the tip ($c_T$) was given by the equilibrium expression

$$c_T = c_g (T_g/T_T)^{2/3} \exp\left[\frac{\alpha E_s (r)}{2kT} \right],$$

(12)

where $c_g$ is the concentration in the gas phase at an appreciable distance from the tip, $T_g$ the temperature of the gas far from the tip, and $\alpha$ the polarizability of the gas atom. Since the gas temperature near the tip is intermediate to $T_g$ and $T_T$, we replaced $T_g$ in the exponential of eq. (12) by simply $T$. Thus, the ion current from a small area on the surface of the tip is given by the expression

$$i[[hkl]] = \frac{\xi^2 \pi c_e r_T^2}{2 T_T} \left( \frac{T_g}{T_T} \right)^{2/3} \int_{r_T \sqrt{x_s}}^{\infty} r^2 vD(r) \exp\left[\frac{-\alpha E^2 (r)}{2kT} \right] dr,$$

(13)

where $\xi$ is the fraction of the total area of the hemisphere ($2\pi r_T^2$) which was sampled in our experiments. Since ionization occurs in a small interval of width $\Delta x$ Å beyond $x_s$, eq. (13) reduced to

$$i[[hkl]] \approx \frac{\xi^2 \pi c_e r_T^2}{2 T_T} \left( \frac{T_g}{T_T} \right)^{2/3} vD(x_s) \exp\left[\frac{-\alpha E_T^2}{2kT} \right] \Delta x.$$

(14)

This equation was evaluated for values of $E_T$ between 2.0 V Å$^{-1}$ and 2.5 V Å$^{-1}$, employing the image potential in $D(x_s)$. $c_g=1.8 \times 10^{14}$ atom cm$^{-3}$, $T_g=100^\circ$K, $T_T=10^\circ$K, $r_T=430$ Å, and $T=50^\circ$K, $40^\circ$K, $60^\circ$K, $80^\circ$K, and $100^\circ$K. A plot of eq. (14) in the form $\log i[[hkl]]$ versus $\log E_T$ was linear with a slope of 31.7 over a range of two orders of magnitude in $i[[hkl]]$ for the above values with $T=50^\circ$K. The calculated values of $i[[hkl]]$ were about a factor of 5 on the low side compared with the experimental values. This agreement with the measured $i[[hkl]]$ is not unreasonable in view of the fact that no correction was made for secondary electrons. This modified version of Gomer’s model also explains the observed increase in slope with decreasing $T_T$ for the total ion current versus $E_T$ characteristic curves sub-section (3.1). That is, the slope of the log $i[[hkl]]$ versus log $E_T$ plot increases as $T$ is decreased. It is felt that the above model is adequate in view of the secondary role that regime I plays in the imaging of atoms.

4.3. The (hkl) and $T_T$ dependence of $i[[hkl]]$

The very complicated behavior of $i[[hkl]]$ in regime II as a function of (hkl) and $T_T$ is one of the most interesting results of the present investigation, and also one of the most difficult to explain quantitatively. The only attempts, to the best of our knowledge, to explain effects of this type are due to Müller et al.[7], Holscher[14] and Van Eekelen[19]. The Müller et al. and Holscher discussions of the problem were qualitative, and Van Eekelen’s treatment was a quantitative one based on a number of simplifying assumptions. In particular, Van Eekelen assumed that $E_s$ was uniform over the surface of the tip, and that all the gas atom velocities were radial. By making these assumptions Van Eekelen removed the patch field nature of the contrast which is basic to all FIM images and thus his calculations are interesting and suggestive, but not definitive.

Let us start by considering* the shape of the characteristic curves at $T_T$ equal to 11 K (see fig. 3) and examine the effect of (hkl) on these curves. First it is essential to realize that $E_T$ is different for each (hkl). This variation in $E_T$ resulted from the field evaporation process which produced a specimen with different values of $r_T$ for different regions of the tip. A high value of $E_T$ implied a large value of $D(x_s)$, and therefore a large value of $i[[hkl]]$. This effect was responsible for the variations in $i[[hkl]]$ from plane-to-plane at a given value of the applied $V_T$. Note that planes belonging to regions B and C imaged at values of $V_T$ which were quite different from the values at which region A imaged. This variation in $E_T$ was also responsible for the fact that there were maxima and minima in the characteristic curves. That is, initially the values of $E_T$ were so low that very little ionization occurred in regions B and C (e.g., see fig. 3). Imaging atoms that arrived in regions B and C, and which were not ionized were there free to migrate to the high field region A

* The present qualitative discussion is an amplification of a number of ideas presented by Holscher[14] as they apply to our experimental results.
where they were ionized. At low $V_T$ the effect of this lateral gas supply\(^\ast\) was to enhance the ion current above the region A planes. The maxima in $i([hkl])$ for the region A planes resulted from the fact that they were deprived of this lateral gas supply once ionization above the region B type planes started to occur. The second maxima in the characteristic curves of the region A planes was a result of the fact that the supply function in regime II was a slowly varying function\(^{19, 24}\) of $E_T$, so that the radial supply of gas atoms compensated for the loss in the lateral supply, and therefore $i([hkl])$ once again increased. The second minima in $i([hkl])$ for some of the region A planes [e.g., (334) and (111)] occurred when ionization started above the region C planes, and hence reduced the lateral gas supply. Similarly, the third maxima in $i([hkl])$ for some of the region A planes took place as a result of the increasing radial supply function with increasing $E_T$. Thus in summary the maxima and minima were interpreted in terms of the combined effects of the regional variation of $E_T$ on the FIM tip, the lateral gas supply, and the slowly increasing field dependent radial supply function.

Next, we consider the effect of $T_T$ on the characteristic curves. The essential physical point involved here is that $dP$ decreases as a result of an increase in $T_T$ [see eq. (2)]. Thus, the range of $E_T$ in regime II over which field ionization occurred was markedly decreased at the higher values of $T_T$ (e.g., compare fig. 3 with fig. 5, or compare the curves at $11^\circ K$ and $86^\circ K$ in fig. 6). Since the value of $E_T$ at which field evaporation occurred had a weak dependence on $T_T$ [e.g., see Chen and Seidman\(^{25}\)] compared to the dependence of $E_0$ on $T_T$, the main effect of an increase in $T_T$ was to decrease the absolute range of $E_T$ in regime II (i.e., the working range decreased with increasing $T_T$). This smaller working range implied that the lateral flux of gas atoms from regions B and C to region A was strongly diminished, because the values of $E_T$ at which the 3 regions imaged were similar at high $T_T$. The fact that the values of $\Delta i([hkl])$ between the remaining maxima and minima decreased was also a result of the smaller lateral flux at the elevated $T_T$. Furthermore, the existence of a maximum in the characteristic curve at $86^\circ K$ is evidence that there was still some lateral flux of gas atoms at this $T_T$, and that the effect was not completely eliminated. In conclusion, we feel that the dependence of $dP$ on $T_T$ was responsible for the observed contrast dependence of the $i([hkl])$ versus $V_T$ curves on $T_T$.

4.4. THE DEPENDENCE OF $i([hkl])$ ON $r_T$ AT CONSTANT $E_T$

The quantity $i([hkl])$ at constant $E_T$ was found to be proportional to a power of $r_T$, which was between 2.3 and 2.9 for the (111) plane, and between 2.5 and 2.8 for the (011) plane. Since we measured $i([hkl])$ from a restricted area on the tip, a reasonable model for this small region is a sphere of radius $r_T$. All the existing theories of the supply function in regime II [Van Eekelen\(^{19}\) and Southon\(^{24}\)] for a freely suspended sphere predicted a dependence which was proportional to $r_T^2$ at constant $E_T$. Thus the stronger dependence observed experimentally on $r_T$ is a good indication that the gas phase was not the only source of imaging gas atoms, but that the shank of the specimen must have also been a source. With respect to this same point we note that Southon and Brandon\(^{9}\) found that the total ion current was proportional to a power of the average tip radius between 2.5 and 3.0. Hence, the existing theories of the supply function have to be modified to include the supply of gas atoms from the shank if absolute ion currents are to be calculated theoretically.

5. Summary and conclusions

(1) The characteristic curves consisted of two distinct regimes. Regime I was linear on a log $i([hkl])$ versus log $V_T$ plot, and had a slope that varied between 27 and 41 which was a function of both $T_T$ and $(hkl)$. A slightly modified form of the existing theory of regime I [Gomer\(^{10}\)] was found to give a reasonable first order fit to the observed data.

(2) Regime II of the characteristic curves exhibited maxima and minima which occurred at different values of $V_T$ for different $(hkl)$ planes. The shapes of the curves were a strong function of $T_T$. Specifically, the effect of increasing $T_T$ was to remove a number of maxima and minima and to smooth out the variations in $i([hkl])$. The shapes of these curves were discussed qualitatively in terms of a patch field model consisting of 3 dominant spatial regions on the surface of the tip. In addition, the explanation also considered the role played by a lateral supply of gas atoms on the surface of the tip, and a slowly increasing field dependent radial supply function. This model provided a reasonable qualitative picture of the characteristic curves. A quantitative analytic model for these processes is still lacking, and the lack of this model represents an unsolved problem in the theory of field ionization and its relationship to contrast in the FIM pattern.

(3) The quantity $V_T$ of the characteristic curve was a linear function of $T_T$ and $(hkl)$ plane. The results for the (011), (111) and (233) are shown in table 2. These observations constituted very strong evidence for the fact that $T_T$ controlled the probability of ionization, and that this probability was a function of both $(hkl)$ and $T_T$.

(4) A revised form of Brandon's model\(^{17}\) was presented to explain the dependence of the cut-off field $(E_0)$ on $T_T$ which was based on the fact that the velocity of the imaging gas atom, when it was ionized, was controlled by

\(^{\ast}\) By lateral we mean that these gas atoms move on the surface of the specimen from one spatial region to another spatial region.
The effect of specimen temperature on the cut-off voltage for the (011), (111) and (233) planes

<table>
<thead>
<tr>
<th>(hkl) plane</th>
<th>Slope (V °K⁻¹)</th>
<th>ΔT</th>
<th>Temperature range (°K)</th>
<th>% Change in Vt</th>
</tr>
</thead>
<tbody>
<tr>
<td>(011)</td>
<td>11.7</td>
<td>51</td>
<td>11 to 62</td>
<td>8.4</td>
</tr>
<tr>
<td>(111)</td>
<td>26.7</td>
<td>75</td>
<td>11 to 86</td>
<td>20.5</td>
</tr>
<tr>
<td>(233)</td>
<td>26.4</td>
<td>63</td>
<td>11 to 74</td>
<td>25.4</td>
</tr>
</tbody>
</table>

Tt. The expression for this dependence was

\[ E_t \approx 2 \left( \frac{m_e}{3h^2} \right)^{\frac{1}{2}} \left( I_t - \phi \right) \left[ \frac{1}{e \ln \frac{\pi}{T \tau}} \right] \left( T_{3/4}/m \right)^{-4}. \]

See section 4.1.1. for the definition of all the parameters. This equation gave a good fit to the data for the (011) plane, but did not have a strong enough dependence on Tt for the data obtained for the (111) and (233) planes. Our inability to fit the data for these planes is due to the lack of a satisfactory potential energy function for the electron above these rather rough atomic planes in the presence of the imaging field.

(5) The quantity \( i[(hkl)] \) at constant \( E_t \) was found to be proportional to a power of \( T_t \) which was between 2.3 and 2.9 for the (111) plane, and between 2.5 and 2.8 for the (011) plane. This result demonstrated that the shank of the FIM made an appreciable contribution to the total supply of imaging gas atoms, and indicated the need for a quantitative model of the supply function which accounts for this source of gas atoms.

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