A quantitative assessment of the three-dimensional microstructure of a $\gamma-\gamma'$ alloy

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Abstract

We present the first quantitative analysis of the three-dimensional microstructure of a $\gamma-\gamma'$ alloy that does not make any a priori assumptions about the $\gamma'$-particle morphologies. The $\gamma'$-particle size distribution is derived using the particle volumes to provide a measure of the particle sizes that is independent of particle morphology and is found to be somewhat broader and to have a lower maximum peak height than the theoretical prediction in the absence of elastic stress. Non-equiaxed $\gamma'$ particles are found to be approximately equally likely to have rod-like, plate-like or mixed-character morphologies. In contrast with theoretical predictions, smaller particles tend to be more non-equiaxed than larger particles. The latter two findings are a result of the attractive interparticle configurational forces induced by stress at the corners of neighbouring particles. By analysing the distribution of particle side lengths, it is concluded that the average thickness of the sheets of $\gamma'$-particles is between 0.4 and 0.6 $\mu$m.

§1. Introduction

During coarsening in stress-free systems the total energy of a two-phase system decreases as its total interfacial area decreases. The coarsening second-phase particles have approximately spherical morphologies, and the cube of the average particle radius increases linearly with time:

$$R^3(t) = K_R t,$$

where $R(t)$ is the average particle radius at time $t$ and $K_R$ is a coarsening rate constant (Lifshitz and Slyozov 1961, Wagner 1961). The particles take on approximately spherical morphologies in an effort to minimize the interfacial area per volume of individual particles.

In many two-phase systems, particularly in solid–solid systems where the interface between the phases is coherent and an elastic misfit is present, the elastic energy is not negligible. In such cases $E_T$, the total of the interfacial and elastic energies, must be considered:

$$E_T = E_E + E_S,$$

where $E_E$ is the total elastic energy and $E_S$ is the total surface energy. $E_E$ consists of the sum of the elastic self-energies of the particles and the elastic interaction energies between particles. As the average particle size increases, the magnitude of $E_E$...
increases relative to the magnitude of $E_S$. Hence, $E_E$ becomes progressively more important as coarsening proceeds and can have a significant effect on the coarsening kinetics and the microstructural evolution, particularly at longer coarsening times. These effects can include changes in the particle morphologies, small particles growing at the expense of larger particles, and particle alignment (Ardell et al. 1966, Su and Voorhees 1996a,b, Wang et al. 1998, Thornton et al. 2003a,b).

The $\gamma-\gamma'$ system is a technically important and widely studied system in which elastic stress is known to affect the coarsening process. The $\gamma'$-precipitate phase has an ordered L1$_2$ structure and a coherent interface with the fcc $\gamma$ matrix phase. There is a dilatational elastic misfit between the $\gamma$ and $\gamma'$ phases due to their different lattice parameters, and the system is elastically anisotropic with elastically soft $\langle 100 \rangle$ crystallographic directions. Particles can take advantage of these elastically soft directions and reduce their total elastic energy by having faces parallel to, and aligning with other particles along, the $\langle 100 \rangle$ directions. Thus, as the particle size increases during coarsening and elastic energy becomes progressively more important, the particle morphologies and spatial alignment gradually evolve from those characteristic of surface energy driven coarsening to those characteristic of elastic energy driven coarsening. At early times when the surface energy is dominant, the particles are spherical and there is no significant particle alignment. As coarsening proceeds and the elastic energy becomes gradually more important, the particles evolve first into cuboidal and then into rod-like or plate-like shapes with faces parallel to the $\langle 100 \rangle$ crystallographic directions. The particles also align with each other along these same $\langle 100 \rangle$ directions (Ardell et al. 1966).

While the effects of elastic energy on the coarsening of $\gamma-\gamma'$ alloys may be most apparent in the microstructure, it has also been reported that the coarsening kinetics are affected by the elastic energy. Experimental and theoretical work has suggested that either the coarsening rate constant or the coarsening time exponent can be time dependent as a result of the growing importance of elastic energy during coarsening (Enomoto and Kawasaki 1989, Leo et al. 1990, Onuki and Nishimori 1991, Doi and Miyazaki 1992). Experimental work by Ardell and various co-workers on $\gamma-\gamma'$ systems has also suggested that the presence of elastic energy can affect the dependence of the rate constant on volume fraction (Ardell 1990, Maheshwari and Ardell 1992, Cho and Ardell 1998, Kim and Ardell 2000). In all these cases a particle radius is used to quantify coarsening. However, in a recent study we find that when coarsening is quantified using interfacial area per volume $S_v$, which is independent of particle morphology, there is no evidence of effects of elastic stress on the coarsening kinetics of Ni–Al $\gamma-\gamma'$ alloys. Also, we conclude that it is not possible to compare accurately the results of our study using $S_v$ to quantify coarsening and previous studies using a particle radius to quantify coarsening. To do so the distribution of both the particle sizes and the particle morphologies in three dimensions would need to be known (Lund and Voorhees 2002a).

There have been some theoretical and experimental studies of the microstructures of $\gamma-\gamma'$ alloys in three dimensions. In single particle calculations the particle morphology has been found to depend only on particle size, with less equiaxed morphologies favoured at larger particle sizes (Johnson et al. 1988, Mueller and Gross 1988, 2000, Thompson and Voorhees 1999). Some multiple-particle calculations have been performed, but they have been limited to a small number of particles or a small number of grid points per particle (Orlikowski et al. 1999, Rubin and Khachaturyan 1999, Vaithanathan and Chen 2000, Chen et al. 2001, Eckert et al. 2002).
2001), and thus an extensive analysis of the particle morphologies and particle size distributions (PSDs) in elastically stressed solids undergoing coarsening has not been performed. A limited number of experimental investigations of three-dimensional $\gamma$–$\gamma'$ microstructures have been performed, including studies using small angle neutron or X-ray scattering (Fahrmann et al. 1995, Mukherji et al. 1999) and deep etching (Yoo et al. 1995). However, a priori assumptions regarding the particles are required to derive specific information from small-angle scattering, while deep etching is limited to partially exposed individual particles. Thus, neither can provide full three-dimensional information on the microstructure.

To obtain full three-dimensional information without making any a priori assumptions regarding the particle morphologies, we performed serial sectioning with digital three-dimensional reconstruction on a model $\gamma$–$\gamma'$ alloy. A qualitative analysis of the resulting microstructure was presented in a previous paper (Lund and Voorhees 2002b), and several conclusions were reached. The $\gamma'$ particles tend to align with each other in parallel two-dimensional sheets. The corners of neighbouring particles, and hence neighbouring particle faces, show a strong tendency to align. As a result, the particle side lengths along the thickness of a sheet tend to be relatively constant. Finally, the particle morphologies appear to be more strongly determined by the interparticle elastic configurational forces than by the sizes of the individual $\gamma'$ particles. In the current paper we extend the analysis of this microstructure and present the first quantitative analysis of an experimentally derived three-dimensional $\gamma$–$\gamma'$ microstructure which does not make any a priori assumptions about the particle morphologies. This provides insights into microstructural evolution in three dimensions during coarsening in $\gamma$–$\gamma'$ systems and hence is a step towards being able to understand fully the coarsening process in elastically stressed solids.

§2. Experimental Procedure

Serial sectioning was performed using a Ni–24.0 wt% Co–5.0 wt% Cr–2.5 wt% Mo–4.0 wt% Al–4.0 wt% Ti alloy. The alloy specimen was solution treated at 1200°C for 135 h, quenched into iced brine and then heat treated for 141.7 h at 1050°C. The heat treatment time and temperature were chosen so that distinctly non-spherical and non-cuboidal particle morphologies were present in the microstructure. At longer heat treatment times on the same alloy the particles were observed to be incoherent, with the particle–matrix interfaces no longer parallel to the $h\{100\}$ crystallographic directions. The $\gamma'$ volume fraction of the alloy following heat treatment was 0.272.

Further details of the serial sectioning technique have been reported in an earlier paper (Lund and Voorhees 2002b). The resulting reconstructed three-dimensional microstructure is a volume measuring $11.33 \mu m \times 11.33 \mu m \times 3.6 \mu m$. The scanning electron microscopy (SEM) images of the serial sections have a resolution of $0.025 \mu m$ pixel$^{-1}$, while the rendered microstructure has a resolution of $0.05 \mu m$ within the imaging plane and a resolution of $0.078 \mu m$ along the polishing direction. This compares with an average $\gamma'$ particle side length of $0.56 \mu m$. Owing to the small $\gamma'$-interparticle separation distances, bridging occasionally occurred between precipitates in the rendered volume. However, as this bridging is not present in the high-resolution SEM images of the serial sections, it is a known artefact of the reconstruction process.
§ 3. Results and Discussion

An example of the reconstructed three-dimensional microstructure is shown in figure 1. The artificial bridging discussed above can be seen in several locations. To analyse quantitatively the individual $\gamma'$ particles the artificial bridging had to be eliminated. This was achieved by splitting the particles into multiple groups before reconstruction on the computer, with neighbouring particles placed into separate groups. When the interfaces are rendered in this manner, they are no longer close enough to their nearest neighbours to cause bridging. Using the surface polygons for each of the individual rendered particles, the centre of mass, the total surface area and the volume can be determined. The morphology of the particles can be described using an aspect ratio but, because the particles are not exactly rectangular parallelepipeds an aspect ratio must first be defined. After an appropriate aspect ratio has been defined, it can also be calculated on the basis of the surface polygons.

Figure 1. A characteristic portion of the three-dimensional rendered microstructure. The frame around the microstructural volume is $7.5 \, \mu m \times 7.5 \, \mu m \times 3.6 \, \mu m$, with the shortest dimension corresponding to the polishing direction during serial sectioning. The $\gamma'$ particles truncated by the top or bottom of the experimental volume have been removed.
3.1. Particle size distribution

The most basic analysis which can be performed using the microstructural data is to investigate the PSD. PSDs are typically plots of frequency versus a normalized particle size $\rho$:

$$\rho = \frac{R}{\bar{R}},$$

where $R$ is a spherical radius for spherical particles or a half-side-length for cuboidal particles and $\bar{R}$ is the average value of $R$ for the system. Although we do not have all spherical or cuboidal particles in the experimental microstructure, we also plot our PSD as a frequency versus a normalized length scale. Specifically, we plot frequency versus $R_{CE}/\bar{R}_{CE}$, with $R_{CE}$ defined as the cubic equivalent half-side-length assuming a cuboidal particle and $\bar{R}_{CE}$ as the average value of $R_{CE}$ in the microstructure. The value of $R_{CE}$ is calculated by

$$R_{CE} = \frac{1}{2} V^{1/3},$$

where $V$ is the measured volume of a given particle. Since we are using the volume to determine $R_{CE}$, the approach is valid for any particle morphology. In addition, if we instead used the radius of a sphere with the equivalent volume, the result would differ only by a constant geometrical factor.

Any particles truncated by or touching the edge of the experimental volume were not included in the PSD; 320 of the 410 particles in the experimental volume met this criteria. Disregarding such particles inherently biases measurement of the PSD, as larger particles are statistically more likely to intersect the edge of the sample volume than smaller particles are. This bias can be removed by taking data only from particles whose centres of mass fall within a measurement volume which is smaller than and located wholly within the experimental volume. The edges of this measurement volume are located at a sufficient distance from the edges of the experimental volume to ensure reasonably that no truncated particles extend into the measurement volume, typically at a distance on the order of the average particle size (Tewari and Gokhale 2001). Owing to the anisotropy of the $\gamma'$-particle morphologies, a conservative distance of half of the maximum $\gamma'$-particle side length in the system was used to separate the edges of the experimental volume and the measurement volume for the PSD; this resulted in the inclusion of 250 particles in the PSD.

The PSD measured from the experimental volume is shown by the histogram in figure 2. This is, to the present authors’ knowledge, the first measured three-dimensional PSD of a $\gamma-\gamma'$ two-phase mixture reported in the literature. Also shown in this figure, by the solid black curve, is the PSD for a stress-free system calculated by Akaiwa and Voorhees (1994) for approximately the volume fraction measured in this experiment. Although 250 particles is a rather small number with which to determine a PSD, certain qualitative features are clear. In particular, the experimental and predicted PSDs are similar, with the experimental PSD being somewhat broader with a lower maximum peak height. Thus, the distribution of particle volumes in the experimental microstructure is surprisingly similar to theoretical predictions despite the presence of elastic stress and the clearly anisotropic particle morphologies.
3.2. Calculation of aspect ratios

In order to provide the most accurate measurements of the particle morphologies, the polishing direction was not parallel to any of the \( <100> \) crystallographic directions. For the purposes of analysis it is useful to rotate the microstructure, or more specifically the individual particles, so that the \( <100> \) crystallographic directions and the \( x, y \) and \( z \) axes used to locate the surface polygons align. As the vertices of a given polygon are known, it is straightforward to calculate its area and normal. A surface orientation distribution can then be obtained using the polygon normals and area. A surface orientation distribution is a contour plot of the percentage of surface area in a system that has a surface normal oriented in a given direction. When deriving the surface orientation distribution it is easiest to define the surface normals in spherical coordinates, with \( \theta \) ranging from 0 to 180 degrees and \( \phi \) ranging from 0 to 360 degrees.

A series of rotations were performed on the microstructure, and the surface orientation distributions before and after these rotations are shown in figure 3. In both distributions there are six peaks, one peak for each \( <100> \) direction. In figure 3(a), before rotation, the peaks are located seemingly at random, but using their \( \phi \) and \( \theta \) orientations we find that the value of the angle separating any two of the peaks is a multiple of 90 degrees when measured on the plane defined by the two orientation vectors. In figure 3(b), after rotation of the coordinate axes, four peaks are now located at \( \phi = 90^\circ \) with \( \theta = 0^\circ, 90^\circ, 180^\circ \) and \( 270^\circ \), and one peak each at \( \phi = 0^\circ \) and \( \phi = 180^\circ \). The first four peaks have well-defined values of both \( \phi \) and \( \theta \), while the last two have well-defined values of \( \phi \) but are dispersed across all \( \theta \). The lack of a well-defined peak at either \( \phi = 0^\circ \) and \( \phi = 180^\circ \) is expected and results from the nature of the spherical coordinate system at the poles, \( \phi = 0^\circ \) or \( 180^\circ \). The locations of the peaks in figure 3(b) confirm that the rotations performed result in the \( <100> \) directions being aligned along the \( x, y \) and \( z \) coordinate axes.

As was reported in the previous paper (Lund and Voorhees 2002b) and can be seen in figure 1, all the particle morphologies in the microstructure are, or are nearly, six-sided rectangular parallelepipeds. Since the particle faces are aligned perpendicular to the \( x, y \) and \( z \) axes, the surface polygons of a chosen particle were binned according to their surface normals. This results in six bins, one each for the two \( x \) faces, one each for the two \( y \) faces, and one each for the two \( z \) faces. For a given particle, polygons with normals of \( \phi < 45^\circ \) or \( \phi > 135^\circ \) belong to the two \( z \) faces of
the particle. The remaining polygons all have values of $\phi$ between $45^\circ$ and $135^\circ$; those with $315^\circ < \theta < 45^\circ$ or $135^\circ < \theta < 225^\circ$ were binned into the two $x$ faces, and those with $45^\circ < \theta < 135^\circ$ or $225^\circ < \theta < 315^\circ$ were binned into the two $y$ faces.

After the polygons were binned by particle face, the centre of mass of each face was calculated. For each particle the side length $s_x$ along the $x$ axis, was then calculated by taking the difference between the $x$ values of the centres of mass of each $x$ face. The side lengths $s_y$ and $s_z$ for each particle were then calculated by

Figure 3. The orientation distribution of the surface polygons (a) before and (b) after rotation of the microstructure. The contours represent the percentage of surface area in the system with normals oriented in a given direction.
equivalent methods. Once the side lengths were known, the aspect ratios could be calculated. First the side lengths of each particle were redefined, with $a$ defined as the maximum side length, $b$ defined as the second-longest side length and $c$ defined as the shortest side length. The aspect ratios were then defined:

$$A = \frac{a}{c},$$

$$B = \frac{b}{c},$$

$$C = 1.$$  \hfill (5)

By this definition, in all cases $A \geq B \geq C$, $C = 1$.

Based on the above definition for the aspect ratio, a particle with a plate morphology has $A = B \gg 1$. More generally, we define a ‘plate-like’ morphology as $A \approx B \gg 1$. Similarly a rod morphology has $A \gg B = 1$, while a ‘rod-like’ morphology has $A \gg B \approx 1$. Equiaxed or ‘cube-like’ morphologies have $A \approx B \approx 1$. The exact boundaries for classifying a particle into one of these categories are ambiguous, and many $(A, B, C)$ do not fit well into any category; they are rectangular parallelepipeds of mixed rod-like and plate-like character. These definitions are shown schematically in figure 4, where $B$ is plotted versus $A$ and examples of the various categories of particle morphology are shown.

There is error involved in the calculation of the aspect ratios. In the case of a perfect rectangular parallelepiped, the method described above would be exact for calculating the particle side lengths and aspect ratios. However, the actual $\gamma'$ particles have curvature at their edges and corners indicative of the resolution of the reconstruction, and faces which are not always perfectly parallel or perpendicular to one another. Thus, the calculated side lengths and aspect ratios are not exact. The volume of the particles can be estimated from the calculated particle side lengths,
The actual volume $V_A$ enclosed by the $\gamma'$ particles can be calculated directly from knowledge of the surface polygons. Thus, the error in the calculated particle edge lengths, and by extension the calculated particle aspect ratios, can be determined by comparing the values of $V_C$ and $V_A$. In nearly all cases the error in $V_C$ was less than $\pm 5\%$, with only eight out of 320 particles not meeting this criterion. Particles with an error in $V_C$ of greater than 5% were not included in the analysis of the particle morphologies which follows.

3.3. Particle morphologies: distribution and size dependence

Calculations of $\gamma'$-particle morphology versus size in two and three dimensions invariably find that larger particles have less equiaxed morphologies than smaller particles (Johnson et al. 1988, Khachaturyan et al. 1988, Thompson et al. 1994, Mueller and Gross 1988, 2000, Thompson and Voorhees 1999, Shneck 2001). Contrary to the predictions of these single-particle calculations, in the qualitative analysis of the experimental microstructure (Lund and Voorhees, 2002b) we concluded that the particle morphologies are not solely determined by particle size. We can now quantitatively investigate the dependence of the particle morphology on particle size using ‘morphology scatter plots’. In these plots the aspect ratio $B$ is plotted as a function of the aspect ratio $A$, as was done in figure 4. Each point on the plots represents an individual $\gamma'$ particle whose aspect ratio is wholly described by its location on the plot, as $C \equiv 1$ by definition. After filtering out particles touching the edge of the measured volume and particles with unacceptably high error in the side length calculations, aspect ratio data were collected for 312 particles. If more data were available, these plots could be made into ‘morphology contour plots’ but, as only 312 total data points are available, scatter plots are more appropriate.

In figure 5 the experimental data have been split into three separate morphology scatter plots labelled Large, Medium and Small. The average particle size in the experimental microstructure is $R_{CE} = 0.26\, \mu m$ and the standard deviation is $\sigma(R_{CE}) = 0.085\, \mu m$. Particles within one half of the standard deviation of the average particle size, $0.218\, \mu m \leq R_{CE} \leq 0.303\, \mu m$, are represented on the scatter plot labelled Medium. Particles larger than this range, $R_{CE} > 0.303\, \mu m$, are represented on the scatter plot labelled Large, and particles smaller than this range, $R_{CE} < 0.218\, \mu m$, are represented on the scatter plot labelled Small. Thus, the particle morphology as a function of particle size is displayed.

In the morphology scatter plot for the large particles in figure 5 most of the particles are clustered in the relatively equiaxed morphology region, with a few particles in the plate-like, rod-like or mixed-character morphology regions. In all cases, $A < 2.75$ and $B < 2.05$ for the larger particles. The medium-sized particles are less clustered in the equiaxed morphology region and are scattered to larger aspect ratios than found with the large particles; $A < 3.40$ (with one outlier) and $B < 2.35$. Again the morphologies with larger aspect ratios are relatively evenly distributed among the plate-like, rod-like and mixed character morphology types. Finally, the small particles show the least clustering in the equiaxed morphology regime and are scattered to the largest aspect ratios of the three size classes; $A < 4.80$ and $B < 3.40$. Once again the morphologies with larger aspect ratios are relatively evenly distributed across the non-equiaxed morphology types.
Figure 5 shows that a variety of particle morphologies are present in the microstructure, and multiple particle shapes are possible for a given particle size class. Further, the non-equiaxed morphologies are relatively evenly distributed among rod-like, plate-like and mixed character morphologies, with smaller size class particles on average tending to be more non-equiaxed than larger size class particles. This result is consistent with our previous qualitative observations (Lund and Voorhees 2002b) but is opposite to the predictions of single-particle equilibrium calculations (Johnson and Cahn 1984, Thompson et al. 1994, Mueller and Gross 1998). Thus, these results indicate that the local particle arrangements and elastic fields must have a significant influence on particle morphology.

3.4. Why do smaller particles tend to be less equiaxed?

Calculations of the equilibrium morphology of an isolated misfitting $\gamma'$ particle predict that the morphology of the particle should become less equiaxed as the volume of the particle increases, with particle morphology changing from spherical, to cuboidal, to rod-like or plate-like with increasing particle volume. Figure 5 shows that the $\gamma'$ particles follow a trend opposite to what is expected on the basis of these calculations alone, with larger particles more equiaxed on average than smaller particles. The volume fraction of $\gamma'$ particles in the system is non-zero, and hence there must be interparticle elastic interactions. As these interactions are not present
in the isolated-particle calculations, it is likely that they are responsible for the less equiaxed smaller particles.

As can be seen in figure 1, interparticle elastic interactions cause the \( \gamma' \) particles to align with each other in two-dimensional sheets of particles. This large-scale alignment results from a negative elastic interaction energy along the (100) crystallographic directions (Johnson and Lee 1979, Khachaturyan 1983, Lund and Voorhees 2002b). While the variation in the elastic interaction energy along the (100) directions accounts for the fact that the \( \gamma' \) particles align along these directions, it cannot provide information on the effects of elastic interactions on particle morphology, as it is a quantity that acts on the entirety of a particle. The effects of interparticle elastic interactions on particle morphology can be illuminated by considering the interparticle configurational forces that are induced by elastic stress (Eshelby 1961, Gurtin and Voorhees 1993, Su and Voorhees 1996a,b). Su and Voorhees found that, for two misfitting particles aligned along crystallographically soft (100) directions, the interparticle configurational forces are repulsive at the centres of the particle faces and attractive at the particle corners. The repulsive forces are responsible for the observed absence of coalescence in the experiments. The presence of attractive forces at the corners leads to the particle alignments and morphologies observed in the three-dimensional microstructure (Lund and Voorhees 2002b), and in microstructures obtained from two-dimensional calculations (Wang et al. 1998, Thornton et al. 2001, 2003a,b). These configurational forces in concert with the sheet-like alignment of the \( \gamma' \) particles result in \( \gamma' \)-particle morphologies that are highly constrained; the edge length along the thickness of a sheet remains relatively constant for all particles within a sheet, while the edge lengths perpendicular to the thickness of a sheet are largely determined by the configuration of neighbouring particles.

Particles aligned within a sheet continue to coarsen to reduce the total interfacial area of the system. However, as per the argument above the particles are locally constrained in their morphologies by configurational forces exerted by the corners of neighbouring particles as coarsening takes place. A simplified schematic diagram of how coarsening could proceed in such a system is presented in figure 6. The schematic diagram begins with two neighbouring cuboidal particles that initially have the same volume and perfectly aligned corners. The particle on the left then grows while the particle on the right dissolves, with the magnitudes of the growth and dissolution rates taken to be similar for both particles. As this occurs, the configurational forces continue to favour alignment of the particle corners, and this restraint leads to relatively constant edge lengths for both particles along the \( x \) and \( z \) directions, and growth and dissolution of the particles primarily along their \( y \) axes. This should not be taken to imply that coarsening occurs by one-dimensional diffusion solely between the two particles. Rather, the coarsening of the particles occurs owing to diffusion throughout the matrix, with the morphologies of the particles locally constrained by interparticle configurational forces. Both particles become non-equiaxed during coarsening; the larger particle has a final aspect ratio of 1.7:1:1, and the smaller particle has a final aspect ratio of 3:3:1. Thus, in this simple schematic representation the introduction of configurational forces favouring alignment of the particle corners causes the smaller particle to become progressively less equiaxed than the larger particle. The degree to which this occurs depends on how closely the configurational forces compel neighbouring particle corners to continue to align during coarsening.
The schematic diagram in figure 6 contains only two particles but, within a sheet, \( \gamma' \) particles have many neighbours that all exert different degrees of interparticle configurational forces. Various combinations of these forces could easily lead to a variety of non-equiaxed plate-like, rod-like and mixed character morphologies, as can be seen from the portion of a sheet shown in figure 7. For example, the particles labelled 1 and 2 are in a configuration very similar to the schematic representation in figure 6. However, it is important to note that the microstructure seen in this experiment is only a snapshot in time; so it is not actually possible to determine which particles were growing and which were dissolving when the microstructure was quenched. As is the case for particle 1, the particles labelled 3 and 4 are both less equiaxed than their relatively equiaxed larger neighbours. In these cases the configurational forces exerted by neighbouring particles have caused particle 3 to evolve into a mixed-character morphology, and particle 4 to evolve into a rod-like morphology. Analogous cases where less equiaxed smaller particles match corners with more equiaxed larger particles can also be seen in the two-dimensional multiple-particle simulations made by both Thornton et al. (2001, 2003a,b) and Wang et al. (1998). These results and those mentioned above all support the hypothesis that the \( \gamma' \) particles with the most non-equiaxed morphologies generally result from the restrictions imposed by the stress-induced interparticle configurational forces exerted by the corners of neighbouring particles.

3.5. Distribution of particle side lengths

The distribution of the non-normalized particle side lengths is shown in figure 8. This distribution includes the values of \( s_x \), \( s_y \) and \( s_z \) for all of the particles in the experimental microstructure, excluding those touching the edge of the experimental volume, those with centres of mass within one half of the maximum \( \gamma' \) side length of the edge of the experimental volume, and those with values of \( V_C \) deviating by more than 5% from \( V_A \). In all, 243 particles fit these criteria. The minimum side length in the distribution is approximately 0.1 \( \mu \text{m} \), which is consistent with the resolution of
Figure 7. A group of particles belonging to the same two-dimensional sheet. Many shapes driven by elastic interactions can be observed. The frame around the particles measures 7.5\,\mu m \times 5.7\,\mu m \times 3.6\,\mu m, with the direction appearing longest in the figure measuring 5.7\,\mu m.

Figure 8. The distribution of the particle side lengths. The values of $s_x$, $s_y$ and $s_z$ for each particle are included.
the experimental technique of 0.05–0.078 µm, and the average side length is 0.56 µm. There is a peak in the distribution in the range 0.4–0.6 µm.

The particles in the experimental microstructure align themselves in two-dimensional sheets, and the particles within these sheets tend to have a relatively consistent side length in the direction of the sheet thickness. This sheet thickness also appears to be relatively consistent from sheet to sheet (Lund and Voorhees 2002b). Since a majority of the particles in the microstructure are constrained in one dimension by a sheet thickness which is relatively constant within a sheet and among different sheets, it would be expected that a definite peak in the distribution of particle side lengths would occur at or near the average sheet thickness. From figure 8, we conclude that the average sheet thickness is in the range 0.4–0.6 µm.

§ 4. Conclusions

We analysed quantitatively the experimentally measured three-dimensional sizes and morphologies of misfitting γ' particles in a model γ–γ' alloy with a γ' volume fraction of 0.272. This work expanded on a previous qualitative analysis of the microstructure (Lund and Voorhees 2002b). We arrive at the following conclusions.

(1) A morphology-independent three-dimensional γ'-PSD can be derived using particle volumes to quantify particle size. Although somewhat broader with a lower peak height, the PSD for the γ' particles was similar to the theoretical prediction in the absence of elastic stress.

(2) Non-equiaxed particles are approximately equally likely to have rod-like plate-like, or mixed-character morphologies.

(3) Smaller particles tend to be more non-equiaxed than larger particles, which is the opposite of what is predicted from theoretical calculations of the equilibrium morphologies of an isolated misfitting particle in an elastically anisotropic system.

(4) The non-equiaxed smaller particles result from constraints imposed on their morphology by neighbouring particles within sheets of γ' particles. These constraints are the result of attractive interparticle configurational forces induced by the stress at the particle corners.

(5) The average sheet thickness lies in the range 0.4–0.6 µm, as indicated by the large number of particle side lengths contained in this range.

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